

DOI: 10.18721/JE.10517
УДК 330.45; 519.833.5

ТЕОРЕТИКО-ИГРОВАЯ МОДЕЛЬ ИНВЕСТИРОВАНИЯ В РАЗВИТИЕ ТЕЛЕКОММУНИКАЦИОННОЙ ОТРАСЛИ

С.А. Черногорский, К.В. Швецов, В.В. Ходырев

Санкт-Петербургский политехнический университет Петра Великого,
Санкт-Петербург, Российская Федерация

Рассмотрен и проанализирован поиск ренты отечественными и иностранными инвесторами телекоммуникационной отрасли. Предложена теоретико-игровая модель, определяющая поведение инвесторов. Получены сравнительные результаты для трех и более инвесторов. Подробно разобрана ситуация с двумя российскими и одним иностранным инвестором телекоммуникационной отрасли России. Цель исследования в том, чтобы найти условия, которые минимизируют потери общественного благосостояния от рентоориентированного поведения российских и зарубежных инвесторов, участвующих в развитии телекоммуникационной отрасли России. Рентоориентированное поведение – это попытка получить экономическую ренту, т. е. часть дохода от фактора производства сверх среднерыночного дохода от него путем манипулирования социальной или политической средой, в которой осуществляется экономическая деятельность, а не за счет создания нового богатства. Такое поведение компаний и организаций тормозит экономический рост, так как приводит к высокому уровню получаемой ренты при очень низком уровне экономической эффективности. В данном случае поиск ренты вредит экономике больше других негативных факторов, так как приводит к отказу от инноваций. Рентоориентированное поведение считается негативным явлением, влекущим значительные потери общественного благосостояния, но во многих рыночных странах большая часть такого поведения является законным, независимо от того ущерба, который оно может нанести экономике. Построены теоретико-игровые модели рентоориентированного поведения в условиях полной и неполной информации в борьбе за ренту. Предложена модель рентоориентированного поведения, когда каждому игроку не безразлично, кому достанется приз, если он сам его не получит. Модель рассмотрена в случаях одинаковой и различной оценки размера ренты игроками. Получены формулы для суммарных равновесных расходов рентоориентированного поведения и индивидуальных равновесных расходов для каждой из отечественных и иностранных фирм при одинаковых и различных оценках размера ренты. Сформулированы выводы в результате использования модели: если члены каждой группы повысят оценку выигрыша других членов своей группы, сумма затрат группы уменьшится; вклад члена группы с общими интересами уменьшится, если увеличится ценность, которую он приписывает выигрышу другого члена группы; с увеличением числа игроков в группе увеличивается вероятность того, что эта группа выиграет, но с другой стороны большее количество игроков с общими интересами означает большую вероятность появления «безбилетников». Проанализирован конкретный пример с тремя игроками – инвесторами в телекоммуникационную отрасль России. Первый и второй игрок являются российскими компаниями, а третий – Intel – иностранной фирмой.

Ключевые слова: теория игр; теоретико-игровая модель; поиск ренты; инвестиции; телекоммуникационная отрасль

Ссылка при цитировании: Черногорский С.А., Швецов К.В., Ходырев В.В. Теоретико-игровая модель инвестирования в развитие телекоммуникационной отрасли // Научно-технические ведомости СПбГПУ. Экономические науки. 2017. Т. 10, № 5. С. 184–194. DOI: 10.18721/JE.10517

A GAME-THEORETIC MODEL FOR INVESTMENTS IN THE TELECOMMUNICATIONS INDUSTRY

S.A. Chernogorskiy, K.V. Shvetsov, V.V. Khodyrev

Peter the Great Saint-Petersburg Polytechnic University, St. Petersburg,
Russian Federation

The rent-seeking behavior of domestic and foreign investors of the telecommunications sector was considered and analyzed. A game-theoretic model determining the behavior of investors was developed. Comparative results for three or more investors were obtained. A situation with two Russian and one foreign investor of the telecom sector of the Russian Federation was examined in detail. The purpose of this article is to find the conditions that minimize the loss in public welfare from the rent-oriented behavior of Russian and foreign investors involved in the development of the Russian telecom industry. Game-theoretic models of rent-seeking behavior under complete and incomplete information in the struggle for rent were constructed. The model of rent-seeking behavior, when all of the players are not indifferent about who will get the prize if they do not receive it themselves, was proposed. The model is analyzed in cases of identical and different estimates of the rent by the players. The formulae for total equilibrium costs of the rent-seeking behavior as well as the individual equilibrium costs for each of the domestic and foreign firms under identical and different estimates of the rent were obtained. The following conclusions were drawn as a result of implementing the model: if the members of each group improved the assessment of the gain of the other members within the group, the total costs of the group will decrease; the contribution of a member of a group with common interests will decrease if the value attributed to the gain of the other group member increases; as the number of players in a group increases the likelihood that the group will win also rises. On the other hand a greater number of players with common interests mean a greater likelihood for so called «free riders» to emerge. An example with three investor players within the Russian telecommunications industry was considered. The first and the second player are Russian companies, and the third player, Intel, is a foreign one.

Keywords: game theory; game-theoretic model; search for rent; investment; telecommunication industry

Citation: S.A. Chernogorskiy, K.V. Shvetsov, V.V. Khodyrev, A game-theoretic model for investments in the telecommunications industry, St. Petersburg State Polytechnical University Journal. Economics, 10 (5) (2017) 184–194. DOI: 10.18721/JE.10517

Introduction. Rent-seeking behavior is an attempt to obtain economic rent, i.e., a part of income from a factor of production above the average market income by means of manipulating the social or political environment in which the economic activity is performed, but not by means of creating new wealth.

Such a behavior of companies and organizations turns out to inhibit economic growth by leading to a high level of the received rent at a very low level of economic efficiency. In this case, the rent-seeking behavior hurts the economy more than other negative factors because it results in failure of innovation [1].

Ultimately, the rent-seeking behavior is regarded as a negative phenomenon entailing considerable losses in public welfare. However, in many free market economies, a large part of

rent-oriented behavior is legal, regardless of the losses it could cause to the economy [2].

An example of rent-seeking in modern economy is lobbying of firms engaged in tenders and auctions for development and production of telecom products in order to increase their market share and/or get preferential treatment over the competitors [3].

The purpose of this article is to find the conditions that minimize the loss in public welfare from the rent-oriented behavior of Russian and foreign investors involved in the development of the Russian telecom industry.

The Government can support or protect a monopolistic position of certain producers, creating or increasing the rent at the expense of buyers and customers. The specified rent turns out to be some kind of prize, stimulating the efforts to obtain it.

The activity aimed at getting the rent created by the Government is called rent-oriented behavior. The term «rent-seeking» was proposed by Ann Krueger in 1974 [4]. The origin of the term is associated mainly with gaining control over land or other natural resources.

The efforts to obtain rent correspond to certain costs. Tullock has shown that from the public viewpoint, considerable resources can be squandered on obtaining the economic rent [5].

According to J. Buchanan et al. [6–8], there are three types of costs associated with rent-oriented behavior:

1. The costs of lobbyists trying to motivate officials responsible for making decisions on creating the rent.

2. The costs of the Government due to obtaining revenues from the corresponding activities and the response to incentives that the rent-seeking behavior induces (because the revenues of the officials that manage the privileges increase by the volume of bribes, there exists some «excessive» competition for appropriate positions; those who wish to take them spend more time and effort) [9].

3. The costs of third parties resulting from the activities of firms or officials involved in rent-seeking behavior (e.g., competition from other interested groups to receive subsidies, tax exemptions, etc.).

The cost of rent-oriented behavior. On the basis of the Tullock model, one can see how the costs of rent-oriented behavior correspond to the rent that stipulates this activity. In particular, we shall obtain the conditions when the costs and the rent are equal. Suppose that n «competitors» are participating in the competition for the rent of size R ; we assume that the probability of obtaining the rent for the participant i is the amount [10]

$$p_i(I_1, \dots, I_n) = \frac{I_i^r}{I_i^r + \sum_{j \neq i} I_j^r},$$

where I_i is the investments of the competitor i in the rent-seeking behavior, r is the parameter which characterizes the investment efficiency. Assuming that all the participants are risk-neutral, the gain of the participant i equals

$$\left(\frac{I_i^r}{I_i^r + \sum_{j \neq i} I_j^r} \right) R - I_i.$$

Provided that the Nash equilibrium of the game is an internal (each participant invests in rent-seeking behavior) and symmetrical one ($I_i = I_j$ for all $i, j = 1, 2, \dots, n$), the value $I_i = I$ turns out to be equal to

$$I = \frac{n-1}{n^2} rR.$$

In a symmetric equilibrium each participant invests in the rent-oriented activity a value of $(n-1)rR/n^2$, if

$$\frac{I^r}{nI^r} R - I \geq 0 \quad (1)$$

and does not invest the assets ($I = 0$) otherwise.

If $r \leq 1$, condition (1) is always true. In other words, if the rent-oriented activity is characterized by non-growing return $r \leq 1$, there is a symmetric equilibrium with positive investments in the rent-seeking behavior. The total investment and share of the investment in the rent are equal to $nI = (n-1)rR/n$ and $\gamma = nI/R = (n-1)r/n$, respectively.

When return of the rent-oriented activity is constant, $r = 1$, the share of the rent offsetting the investments with $n = 2$ equals 1/2. If the number of the participants increases, the share of the rent offsetting the investment approaches 1.

Let us consider the case of increasing return, $r > 1$. If $r > 2$, condition (1) is not true at any n , because there is no symmetric equilibrium in pure strategies. If $r = 2$, condition (1) is true only when $n = 2$. In this case, each of the two parties invests exactly half of the potential rent in the rent-oriented activity so that the total investment equals exactly the rent [11].

If $r < 2$, the equilibrium can exist when n is greater than 2. For example, if $r = 1.5$, condition (1) holds true when $n = 3$. If $n = 2$, the value of total investment amounts to two thirds of the rent, and when $n = 3$ it exactly equals the rent. We should note that until the expected gain from rent-oriented activity proves to be positive, there is an incentive attracting additional participants in the game. Thus, free entrance and constant return make a well-predicted «rent-dissipation» the result of competition for the rent.

Incomplete information in the model of rent-competition. We have examined the models of rent-oriented behavior, assuming that the

decisions were made by the participants with all the information available [12]. Now we are going to abandon the assumption that the participants competing for rent are fully informed. We will consider the situation when the number of the competing participants equals two, whereas the first participant does not know the prize evaluation of the second participant. Therefore, according to the terminology of the game theory, player 1 does not know the type of player 2. Let us also assume that the competition for rent is described by the Stackelberg model, where player 1 (leader) makes the decision first, and player 2 decides knowing the decision of the leader. In order to make the assumption of incomplete information true, we shall suppose that estimates of the prize are different with different players. Finally, we assume that the technology of the rent-oriented activity is characterized by constant return $r = 1$.

Let us assume that the prize estimation of player 2 can take two values: either v_1 , or v_h ($v_2 \in \{v_1, v_h\}$), where $v_h > v_1$. In terms of game theory, it means that player 2 can be of two types. Suppose that player 2 can be the type of player v_h with probability q , and, accordingly, the type of player v_1 with probability $(1 - q)$. It is easy to see that the equilibrium strategy in the game goes as follows:

$$x_2^*(x_1) = \begin{cases} \max\{0, (v_h x_1)^{1/2} - x_1\}, & \text{if } v_2 = v_h, \\ \max\{0, (v_1 x_1)^{1/2} - x_1\}, & \text{if } v_2 = v_1. \end{cases} \quad (2)$$

Player 1 must choose x_1 so as to maximize the expected payoff, i.e., to maximize the following expression:

$$E[U_1(x_1)] = q v_1 \Pr(\text{where } v_2 = v_h) + (1 - q) v_1 \Pr(\text{where } v_2 = v_1) - x_1.$$

In the beginning we suppose that the costs of player 2 are positive. We substitute the expression for $x_2(x_1)$ in equation (2). Then we obtain that

$$E[U_1(x_1)] = q \frac{v_1 x_1}{(v_h x_1)^{1/2}} + (1 - q) \frac{v_1 x_1}{(v_1 x_1)^{1/2}} - x_1.$$

The decision of player 1 is characterized by the first-order condition:

$$\frac{v_1 x_1^{*-1/2}}{2} [q v_h^{-1/2} + (1 - q) v_1^{-1/2}] = 1.$$

This expression determines the costs of player 1 unambiguously as:

$$x_1^* = \frac{v_1^2}{4} [q v_h^{-1/2} + (1 - q) v_1^{-1/2}]^2.$$

Besides, we know that if $x_1^* = v_1^2/4 [q v_h^{-1/2} + (1 - q) v_1^{-1/2}]^2 \geq v_1$, a player 2 who does not appreciate the prize chooses not to participate in the competition. This happens when $v_1 \geq 2 v_1^{1/2} [q v_h^{-1/2} + (1 - q) v_1^{-1/2}]^{-1}$. In this case the best answer of player 2 in the equilibrium will be abandoning the investment. Then player 1 would maximize the following expression:

$$E[U_1(x_1)] = q \frac{v_1 x_1}{(v_h x_1)^{1/2}} + (1 - q) v_1 - x_1$$

provided that $x_1 \geq v_1$.

When the constraint is not important, the maximum of the first order condition is written as $v_1 q (v_h x_1)^{-1/2} / 2 = 1$. Here we get the value of optimal costs for player 1 as $x_1^* = \max\{v_1, q^2 v_1^2 / (4 v_h)\}$.

If $x_1^* = q^2 v_1^2 / (4 v_h) \geq v_1$, then the best answer of player 2, regardless of the type, will be zero cost. This condition is fulfilled when $v_1 \geq 2 v_h / q$, in this situation player 1 will choose the amount of the costs as v_h .

Let us put all the results together:

$$x_1^* = \begin{cases} v_1^2/4 [q v_h^{-1/2} + (1 - q) v_1^{-1/2}]^2, \\ \text{if } v_1 < 2 v_1^{1/2} [q v_h^{-1/2} + (1 - q) v_1^{-1/2}]^{-1}, \\ \max\{v_1, q^2 v_1^2 / (4 v_h)\}, & \text{if } 2 v_1^{1/2} [q v_h^{-1/2} + \\ + (1 - q) v_1^{-1/2}]^{-1} \leq v_1 < 2 v_h / q, \\ v_h, & \text{if } v_1 \geq 2 v_h / q. \end{cases}$$

That is, if the probability q is low, then the equilibrium choice of player 1 is close to the solution with full information. The same is true if q is close to 1. Moreover, x_1^* is a non-decreasing function of v_1 and q and non-increasing function of v_h and v_1 .

A competition-for-rent model, when each player cares about who will get the prize if the player does not receive it. In seeking-for-rent situations the participants are often not indifferent of who will get the rent, even when it is not themselves [13].

In other words, the rent has a well-known characteristic of a public good, i.e., is a distributed public good [14]. In order to implement the idea of such an assessment, we shall consider the rent evaluations as a vector. The rent estimation of each player is a vector $\mathbf{v}_i = (v_{i1}, v_{i2}, \dots, v_{in})$, where v_{ij} is the value for player i if player j gets the rent. The probability that a player will get the rent equals the share costs in the total expenditure, i.e., the probability that player i will receive the rent, $p_i(\mathbf{x})$, equals x_i/s , where $s = \sum_{j=1}^n x_j$, $\mathbf{x} = (x_1, x_2, \dots, x_n)$ and x_i denotes the costs of player i .

Let $\mathbf{p}(\mathbf{x})$ be the vector of corresponding probabilities $\mathbf{p}(\mathbf{x}) = (p_1(\mathbf{x}), p_2(\mathbf{x}), \dots, p_n(\mathbf{x}))$. The expected profit of each player depends not only on how the players assess their own gain (in case they gain), but also on how they evaluate the gain of the other players. If we look at a game of, say, three players, then the expected gain of the first player is the sum of the rent value in case this player wins multiplied by the probability of winning the game, plus the gains of the other players for the cases of their winning multiplied by the corresponding probabilities, minus the costs of the first player. In mathematical symbols it looks as follows

$$U_1(x_1, x_2, x_3) = v_{11} \frac{x_1}{x_1 + x_2 + x_3} + v_{12} \frac{x_2}{x_1 + x_2 + x_3} + v_{13} \frac{x_3}{x_1 + x_2 + x_3} - x_1.$$

We can express the expected profit of player i in the competition of n players more concisely using matrix notation $U_i(\mathbf{x}) = \mathbf{v}_i^T \mathbf{p}(\mathbf{x}) - x_i$. These utility functions assume that the players are risk-neutral [15].

We will define an $n \times n$ matrix composed of vectors of assessments $\mathbf{v}_1^T, \mathbf{v}_2^T, \dots, \mathbf{v}_n^T$. The Nash equilibrium can be obtained by finding the solutions of n equations that are the first-order terms, which can be written in matrix form as follows:

$$\begin{pmatrix} v_{11} & v_{12} & \dots & v_{1n} \\ v_{21} & v_{22} & \dots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1} & v_{n2} & \dots & v_{nn} \end{pmatrix} \begin{pmatrix} \frac{x_1}{s} \\ \frac{x_2}{s} \\ \vdots \\ \frac{x_n}{s} \end{pmatrix} - \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{pmatrix}.$$

If we assume that $\mathbf{U}(\mathbf{x}) = (U_1, U_2, \dots, U_n)^T$ is the vector of gains of the players, we can summarize the above expression, using matrix notation $\mathbf{U}(\mathbf{x}) = \mathbf{V}\mathbf{p}(\mathbf{x}) - \mathbf{x}$.

Let us define the condition of the maximum of the first order for the expected profit of the first player using the mathematical analysis, then:

$$\frac{\partial U(x_1, x_2, \dots, x_n)}{\partial x_1} = \frac{v_{11}s - v_{11}x_1}{s^2} - \frac{v_{12}x_2}{s^2} - \dots - \frac{v_{1n}x_n}{s^2} - 1 = 0$$

or

$$\frac{(v_{11} - v_{12})x_2}{s^2} + \dots + \frac{(v_{11} - v_{1n})x_n}{s^2} = 1$$

In the same way we can find the first-order condition of maximized expected utility of each player and summarize the results, getting the following expression:

$$\begin{pmatrix} 0 & v_{11} - v_{12} & \dots & v_{11} - v_{1n} \\ v_{22} - v_{21} & 0 & \dots & v_{22} - v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{nn} - v_{n1} & v_{nn} - v_{n2} & \dots & 0 \end{pmatrix} \begin{pmatrix} \frac{x_1}{s^2} \\ \frac{x_2}{s^2} \\ \vdots \\ \frac{x_n}{s^2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}.$$

To make it a bit more concise we can simplify the expression above to matrix notation $(\mathbf{J}_{n \times n} - \mathbf{V}) \frac{\mathbf{x}}{s^2} = \mathbf{1}_n$, where $\mathbf{J}_{n \times n}$ is an $n \times n$ matrix where row i consists of v_{ii} and $\mathbf{1}_n$ is an $n \times 1$ vector.

We can analyze how the results will change as the number of players increases. To do this, we add some players with interests similar either to the interests of players 1 and 2 or the interests of player 3 from the previous example. We will consider mobile phone manufacturers, which are divided into two groups: domestic and foreign ones. We suppose that there are n manufacturers who have common interests with players 1 and 2 and m manufacturers that have the same interests as player 3. We can consider a set of players consisting of two parts. Domestic manufacturers are 1, 2, ..., n and foreign ones are $n+1, n+2, \dots, n+m$.

Payments to the firms in these two groups can be characterized by many players that share

common interests and the value they attribute to the winning company of the group. For domestic manufacturers each player estimates the rent as 1 in case of winning and γ if some other domestic producer wins. For a foreign manufacturer the rent is estimated as 1 in the case of winning and δ if another foreign manufacturer wins. The expected utility, which the players want to maximize, can be written as follows [16]:

$$\begin{pmatrix} 1 & \gamma & \dots & \gamma & 0 & \dots & 0 & 0 \\ \gamma & 1 & \dots & \gamma & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma & \gamma & \dots & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 1 & \delta & \dots & \delta \\ 0 & 0 & \dots & 0 & \delta & 1 & \dots & \delta \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & \delta & \delta & \dots & 1 \end{pmatrix} \begin{pmatrix} \frac{x_1}{s} \\ \vdots \\ \frac{x_n}{s} \\ \frac{x_{n+1}}{s} \\ \vdots \\ \frac{x_{n+m}}{s} \end{pmatrix} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \\ x_{n+1} \\ \vdots \\ x_{n+m} \end{pmatrix} = \begin{pmatrix} U_1 \\ \vdots \\ U_n \\ U_{n+1} \\ \vdots \\ U_{n+m} \end{pmatrix}.$$

The first-order condition is determined in the same manner as in the previous example. The matrix $(\mathbf{J}_{(n+n) \times (n+m)} - \mathbf{V})$ can be inverted for any $\gamma, \delta \neq 1$ and $n, m \geq 1$. Using the notation $\mathbf{I}_{k \times k}$ for the $k \times k$ identity matrix, we can get a solution for the vector of equilibrium costs \mathbf{x}^* .

$$\mathbf{x}^* = (s^*)^2 (\mathbf{J}_{(n+n) \times (n+m)} - \mathbf{V})^{-1} \mathbf{1}_{(n+m)}.$$

For the above expression we have

$$(\mathbf{J}_{(n+n) \times (n+m)} - \mathbf{V})^{-1} = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix}$$

with W_{ij} defined below.

$$\begin{aligned} W_{11} &= -\frac{1}{(1-\gamma)} \times \\ &\times \left[\mathbf{I}_n - \frac{(1-\gamma)(1-\delta)(m-1) - m}{(n-1)(m-1)(1-\gamma)(1-\delta) - nm} \mathbf{J}_{n \times n} \right], \\ W_{12} &= \frac{-1}{(n-1)(m-1)(1-\gamma)(1-\delta) - nm} \mathbf{1}_{n \times m}, \\ W_{21} &= \frac{-1}{(n-1)(m-1)(1-\gamma)(1-\delta) - nm} \mathbf{1}_{m \times n}, \\ W_{22} &= -\frac{1}{(1-\delta)} \times \\ &\times \left[\mathbf{I}_m - \frac{(1-\gamma)(1-\delta)(n-1) - n}{(n-1)(m-1)(1-\gamma)(1-\delta) - nm} \mathbf{J}_{m \times m} \right]. \end{aligned}$$

Now we can deduce an expression both for total equilibrium costs s^* and individual equilibrium costs. We will use x_d^* to denote the equilibrium costs for each of the domestic firms (players 1, 2, ..., n) and x_f^* for each of the foreign manufacturers (players $n+1, n+2, \dots, n+m$). Then,

$$\begin{aligned} s^* &= \frac{nm - (1-\gamma)(1-\delta)(n-1)(m-1)}{n[m - (1-\delta)(m-1)] + m[n - (1-\gamma)(n-1)]}, \\ x_d^* &= (s^*)^2 \left[\frac{m - (1-\delta)(m-1)}{nm - (1-\gamma)(1-\delta)(n-1)(m-1)} \right], \\ x_f^* &= (s^*)^2 \left[\frac{n - (1-\gamma)(n-1)}{nm - (1-\gamma)(1-\delta)(n-1)(m-1)} \right]. \end{aligned}$$

These expressions can be solved for x_d^* and x_f^* in terms of game properties. It should be noted that each firm will make a contribution to the equilibrium.

Based on the results obtained for the equilibrium costs, we can deduce some simple comparative results. Firstly, if the members of each group increase the assessment of the gain of other members within the group (γ or δ increases) then the total costs will decrease, i.e., $\partial s^* / \partial \gamma, \partial s^* / \partial \delta < 0$. In addition, the contribution of a group member with common interests will decrease as the gain assessment attributed to another group member increases, i.e., $\partial x_d^* / \partial \gamma, \partial x_f^* / \partial \delta < 0$. One of the ways to see what happens if we change the parameters is to consider the relationship $nx_d^* / mx_f^* = (nm - (1-\delta)(m-1)n) / (nm - (1-\gamma)(n-1)m)$ which is the ratio of the probability that the domestic manufacturer wins to the probability that the foreign one does. It is easy to see that this ratio will increase if foreign manufacturers attribute a higher value to each of them winning the rent (so that δ increases) or if domestic manufacturers give a lower value to each of them winning the rent (γ decreases). Moreover, as the number of players in a group increases, simultaneously increases the likelihood that the group will win. However, an increasing number of players with common interests mean a greater likelihood for «free riders» to pop up, when there are more players with interests that are the same as yours. This increases your chance of winning, as long as the interests of players are not identical [17].

The difference in the rent estimates. Now let us analyze a game where players have their own estimates of the prize. We shall consider three investor players of the Russian telecom industry. Let the Almaz-Antey Concern be the first player, JSC RTI GROUP (RTI) the second one, and Intel the third player. We denote the gain estimates of the players as follows: v_{11} for Almaz-Antey, v_{22} for RTI and v_{33} for Intel. However, the gain estimates of winning from each other for Almaz-Antey and RTI make up a share of their own estimate. We assume that the share is equal for the two. Then RTI estimates the gain of Almaz-Antey as γv_{22} , while Almaz-Antey attributes the value of γv_{11} to the gain of RTI. In this case, Intel estimates the gain of Almaz-Antey or gain of RTI as zero. We can take these asymmetric rents as a result of various circumstances of the players. For example, if the stock price of RTI is more sensitive to foreign competition, then the company may evaluate the protectionist legislation higher than Almaz-Antey. For simplicity, we will analyze only this competition of three players. The matrix of estimates is presented in the following form:

$$V = \begin{pmatrix} v_{11} & \gamma v_{11} & 0 \\ \gamma v_{22} & v_{22} & 0 \\ 0 & 0 & v_{33} \end{pmatrix}.$$

Determining the first-order conditions in this task is somewhat more difficult than in the previous examples, because we assumed that $v_{ii} \neq v_{jj}$. However, the same logic still holds, and the solution, provided that all players participate, gives the following equilibrium costs:

$$x_1^* = \frac{2v_{11}v_{22}v_{33} [v_{11}v_{33} - v_{22}v_{33} + v_{11}v_{22}(1-\gamma)]}{[v_{11}v_{22} + v_{22}v_{33} + v_{11}v_{33} + v_{11}v_{22}\gamma]^2 (1-\gamma)}, \quad (3)$$

$$x_2^* = \frac{2v_{11}v_{22}v_{33} [v_{22}v_{33} - v_{11}v_{33} + v_{11}v_{22}(1-\gamma)]}{[v_{11}v_{22} + v_{22}v_{33} + v_{11}v_{33} + v_{11}v_{22}\gamma]^2 (1-\gamma)}, \quad (4)$$

$$x_3^* = \frac{2v_{11}v_{22}v_{33} [v_{22}v_{33}(1-\gamma) + v_{11}v_{33}(1-\gamma) - v_{11}v_{22}(1-\gamma)^2]}{[v_{11}v_{22} + v_{22}v_{33} + v_{11}v_{33} + v_{11}v_{22}\gamma]^2 (1-\gamma)} \quad (5)$$

$$s^* = \frac{2v_{11}v_{22}v_{33}}{[v_{11}v_{22} + v_{22}v_{33} + v_{11}v_{33} + v_{11}v_{22}\gamma]^2 (1-\gamma)}. \quad (6)$$

It is important to note that the above costs form an equilibrium only with the involvement of all players. Of course, if the numerator of Eqs. (3)–(6) is negative, the players will decide not to play. It turns out that the same conditions that stipulate strictly positive contributions in Eq. (3)–(6) have a very beautiful interpretation. These conditions answer the following question: «If two players are already in the game, then when does the third player choose to carry the cost of participation in the game?» We also would like to know whether the order of the players entering the game influences the final set of players, and under what conditions all three players will participate in this game.

We suppose first that Almaz-Antey and RTI are playing against each other whereas Intel has to decide whether to join the competition for the rent. Of course, Intel will only participate if its marginal expected profit is positive. That is, Intel wants to participate in the competition, if $\partial U_3(x_1, x_2, x_3)/\partial x_3 > 0$. In this example, the condition of Intel's participation goes as following:

$$\begin{aligned} \frac{\partial U_3(x_1, x_2, x_3)}{\partial x_3} &= \frac{(x_1 + x_2)v_{33}}{s^2} - 1 = \\ &= \frac{(s - x_3)v_{33}}{s^2} - 1 > 0. \end{aligned}$$

Since we evaluated the derivative with $x_3 = 0$, then s is the sum of the equilibrium cost of both Almaz-Antey and RTI, if they play with each other $s = v_{11}v_{22}(1-\gamma)/(v_{11} + v_{22})$. Therefore, we obtain that Intel wants to participate if:

$$v_{33} > v_{11}v_{22}(1-\gamma)/(v_{11} + v_{22}).$$

We should note that the condition that gives a positive contribution for Intel in equation (5) is the same that gives the company a positive marginal expected utility. It is also interesting to note that as the mutual evaluation of the gain of Almaz-Antey and RTI increases (γ increases), the minimum value of the prize required by Intel to participate is reduced. In the case when the prize turns out to be a pure public good for Almaz-Antey and RTI (γ approaches 1), Intel chooses to participate in the game. The degree of competition of Almaz-Antey and RTI will decline as their mutual evaluation of each other's gain increases (γ increases) until ultimately the

companies will participate in the game as a single player. In this case, Intel joins the competition with a lower gain evaluation, and if the prize is a pure public good for two other players, Intel will always participate.

If we consider the conditions of entry for RTI or Almaz-Antey, the analysis is only slightly more complicated. We assume that the Almaz-Antey and Intel are leading the game. When does RTI want to participate? Using the same arguments as before, we get that RTI will decide to make a contribution if it increases its expected utility or if $\partial U_2(x_1, x_2, x_3)/\partial x_2 > 0$. This requirement means that the first-order condition goes as follows:

$$\frac{\partial U_2(x_1, x_2, x_3)}{\partial x_2} = \frac{v_{22}(1-\gamma)x_1}{s^2} + \frac{v_{22}x_3}{s^2} - 1 > 0.$$

This expression can be simplified by multiplying it by s and regrouping the summands.

This will give $\frac{v_{22}(s-x_2)}{s} + \frac{\gamma v_{22}x_1}{s} > s$. Almaz-

Antey and Intel will implement their equilibrium strategies in the game of two players, where Almaz-Antey evaluates the prize as v_{11} and Intel evaluates the prize as v_{33} . The equilibrium contributions in the game of two players are $x_1 = v_{11}^2 v_{33} / (v_{11} + v_{33})^2$, $x_3 = v_{11} v_{33}^2 / (v_{11} + v_{33})^2$, respectively, and $s = v_{11} v_{33} / (v_{11} + v_{33})$. If we make the substitution and evaluate the derivative with $x_2 = 0$, then we can deduce the condition for participation of RTI in the game:

$$v_{22} > \frac{v_{11} v_{33}}{v_{11}(1-\gamma) + v_{33}}. \quad (7)$$

It is again the same condition which provides positive costs in equation (4). If RTI evaluates the gain of Almaz-Antey as zero ($\gamma = 0$), then it will be again the same result obtained earlier. While considering condition (7) we see that in order to participate RTI has to evaluate the gain higher as the estimates of Almaz-Antey or Intel (v_{11} or v_{33}) are increasing or if both Almaz-Antey and RTI will evaluate each other's gains higher (γ increases). It is intuitively clear that Almaz-Antey will make a greater contribution to the game of two players if its own gain estimate increases (v_{11}) while RTI will be able to take a «free ride». If RTI evaluates the gain of Almaz-Antey higher (γ increases), then RTI would make a smaller contribution, since it gets a higher expected utility

from the contribution of Almaz-Antey. Moreover, the increase in gain estimates by Intel (v_{33}) will make the contribution of RTI less profitable, so it must assess the gain higher prior to choosing to take part in the game. Let us examine what would happen if RTI and Almaz-Antey's estimates of winning from each other are getting close to the estimates of their own win (γ approaches 1). In this case, RTI chooses to participate in the competition only if its estimate exceeds that of Almaz-Antey.

We should note that at least two players are always active in the game, with no dependence on the order in which players enter the competition. The conditions for switching the players from non-participation to participation are identical to the conditions determining that their expenditure is greater than zero if they are already in the game [18].

If we assume that all players are active and this situation will stay the same with small changes of the parameters, then we can draw some conclusions regarding the game. It is not surprising that if Almaz-Antey and RTI attributed higher importance to winning from each other (γ increases), then their contributions and overall costs (x_1^* , x_2^* and s^*) would decrease while Intel's cost x_3^* would increase. It will be clear if we consider the probability of Almaz-Antey or RTI winning divided by the probability of Intel winning:

$$\frac{x_1^* + x_2^*}{x_3^*} \frac{2v_{11}v_{22}}{v_{33}(v_{11} + v_{22}) - v_{11}v_{22}(1-\gamma)}.$$

It is obvious that the probability of Almaz-Antey or RTI winning decreases as the values which they attribute to each other's win increase (γ increases) or as Intel's prize assessment increases (v_{33} increases). An increase in «self-assessment» (v_{11} or v_{22}) will also increase the ratio, indicating that Intel's win is less probable.

An example. Now we shall evaluate the behavior of the three players assuming that Almaz-Antey estimates its proposal as 1. However, the company prefers for RTI's proposal to be accepted rather than Intel's [19]. Almaz-Antey evaluates RTI's win as $\gamma < 1$. RTI also evaluates its proposal as 1, and the company prefers for Almaz-Antey's proposal to be accepted rather than Intel's. Intel also evaluates

its proposal as 1 and the company wishes that only its proposal were accepted, while the other two have no value for Intel. Let us assume that Almaz-Antey and RTI evaluate each other's proposals equivalently, so the expected profit of these firms can be expressed as follows:

$$\begin{pmatrix} 1 & \gamma & 0 \\ \gamma & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{x_1}{s} \\ \frac{x_2}{s} \\ \frac{x_3}{s} \end{pmatrix} - \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix}.$$

Based on first-order conditions we obtain:

$$\begin{pmatrix} x_1^* \\ x_2^* \\ x_3^* \end{pmatrix} = \frac{(s^*)^2}{2(1-\gamma)} \begin{pmatrix} -1 & 1 & 1-\gamma \\ 1 & -1 & 1-\gamma \\ 1-\gamma & 1-\gamma & -(1-\gamma)^2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

We can find the values of s^* , x_1^* , x_2^* and x_3^* from the above equation:

$$\begin{aligned} s^* &= 2 / (3 + \gamma), \\ x_1^* &= x_2^* = 2 / (3 + \gamma)^2, \\ x_3^* &= 2(1 + \gamma) / (3 + \gamma)^2. \end{aligned}$$

Thus, the result is that if Almaz-Antey and RTI give a greater value to each other winning the competition, then there will be less overall activity in the struggle for rent ($\partial s^* / \partial \gamma < 0$) and lower costs for both companies ($\partial x_1^* / \partial \gamma, \partial x_2^* / \partial \delta < 0$). However, Intel pays more as γ increases ($\partial x_3^* / \partial \gamma > 0$). In other words, if Almaz-antey and RTI were in greater accordance on the legislation, then their individual contributions in the equilibrium would decrease, whereas the Intel's contribution would increase. The increase of Intel's contribution is smaller than the overall reduction in the contributions from both Almaz-Antey and RTI. This shows that as RTI and Almaz-Antey are becoming closer in the estimates, the competition for Intel is getting more expensive. The fact that rent is an imperfect public good means lower costs for rent competition. In addition, the player whose winning is socially undesirable will spend more and the probability of this player's winning in the equilibrium increases.

Conclusion

1. The game-theoretic models of rent-seeking behavior with complete and incomplete information in the competition for rent were constructed.

2. The model of rent-seeking behavior when all of the players are not indifferent about who will get the prize if they do not receive it themselves was proposed. The model is analyzed in cases of identical and different estimates of the rent by the players.

3. The formulae for total equilibrium costs of the rent-seeking behavior as well as the individual equilibrium costs for each of the domestic and foreign firms under identical and different estimates of the rent were obtained.

4. Some conclusions were drawn as a result of the model implementation:

- if the members of each group improved the assessment of the gain of other members within the group, the total costs of the group will decrease;
- the contribution of a member of a group with common interests will decrease if the value attributed to the gain of another group member increases;
- as the number of players in a group increases the likelihood that the group will win also increases. On the other hand, a greater number of players with common interests mean a greater likelihood for so called «free riders» to emerge.

5. An example with three players-investors within the Russian telecommunications industry was considered. The first and the second player (Almaz-Antey and RTI) are Russian companies, and the third player, Intel, is a foreign one.

The result is that if Almaz-Antey and RTI give a greater value to each other's wins then their costs in the competition for rent will be lower. This concerns both the total cost and the individual cost of the Russian companies. In other words, if Almaz-Antey and RTI were in greater accordance on the legislation, then their individual contributions in the equilibrium would decrease, whereas Intel's contribution would increase. This shows that as RTI and Almaz-Antey are becoming closer in the estimates for the best proposal, the competition is getting more expensive for Intel.

In this case, the probability of winning an auction for creating new telecommunications equipment for domestic companies is higher than that for foreign ones.

СПИСОК ЛИТЕРАТУРЫ

- [1] **Murphy K.M., Shleifer A., Vishny R.W.** Why is Rent-Seeking So Costly to Growth? // The American Economic Review. 1993. No. 83(2). P. 409–414.
- [2] **Левин М., Сатаров Г.** Рентоориентированная Россия // Вопросы экономики. 2014. № 01. С. 61–77.
- [3] **Feenstra R., Taylor A.** International Economics. New York: Worth Publishers, 2008. ISBN 978-0-7167-9283-3
- [4] **Krueger A.** The Political Economy of the Rent-Seeking Society // American Economic Review. 1974. Vol. 64, no. 3. P. 291–303.
- [5] **Tullock G.** The Welfare Costs of Tariffs, Monopolies, and Theft // Western Economic Journal. 1967. Vol. 5, no. 3. P. 224–232.
- [6] **Buchanan J., Tullock G., Tollison R.** Toward a Theory of the Rent-Seeking Society. Texas A&M University, Texas & M University Economics Series, 1980. 367 p.
- [7] **Tullock G.** Efficient rent-seeking // Buchanan J.M., Tullock G. Tollison R.D. Toward a Theory of the Rent-Seeking Society. Texas A&M. University Press, 1980.
- [8] **Corcoran W.J.** Long-run equilibrium and total expenditures in rent-seeking // Public Choice. 1984. No. 43 (1). P. 89–94.
- [9] **Leeson P.T.** The Invisible Hook: The Hidden Economics of Pirates // Princeton University Press. 2009. P. 191.
- [10] **Бусыгин В.П., Желободько Е.В., Цыплавков А.А.** Микроэкономика – третий уровень. Новосибирск: СО РАН, 2003. 704 с.
- [11] **Thijssen J.J.J., Huisman K.J.M., Kort P.M.** Symmetric equilibrium strategies in game theoretic real option models // Journal of Mathematical Economics. 2012. No. 48 (4). P. 219–225.
- [12] **Nitzan S.** Collective Rent Dissipation // Economic Journal. 1991. Vol. 101, no. 402. P. 1522–1534.
- [13] **Mokyr J., John V.C. Nye.** Distributional Coalitions, the Industrial Revolution, and the Origins of Economic Growth in Britain // Southern Economic Journal. 2007. No. 74(1). P. 50–70
- [14] **Baik K., Lee S.** Collective Rent Seeking with Endogenous Group Sizes // European Journal of Political Economy. 1997. Vol. 13, no. 1. P. 121–130.
- [15] **Cavusoglu H., Raghunathan S., Yue W.T.** Decision-Theoretic and game-theoretic approaches to IT security investment // Journal of Management Information Systems. 2008. No. 25 (2). P. 281–304.
- [16] **Черногорский С.А.** Рентоориентированное поведение российских и иностранных автопроизводителей // Интеграция экономики в систему мирохозяйственных связей: сб. науч. тр. X Междунар. Конф. СПб.: Изд-во СПбГПУ, 2005. С. 181–185.
- [17] **Pasour E.C.** Rent Seeking: Some Conceptual Problems and Implications // The Review of Austrian Economics. 1987. Vol. 1. P. 123–145.
- [18] **Шаститко А.Е.** Экономическая теория институтов. М.: Теис, 1997.
- [19] **Schenk R.** «Rent Seeking». CyberEconomics. Archived from the original on January 3, 2006. Retrieved 2007-02-11.
- [20] **Матвеев В.Д., Королев А.В., Бахтин М.А.** Игровые равновесия в сетевых моделях с разной степенью зависимости от среды // Государство и бизнес. Современные проблемы экономики: матер. IX Междунар. науч.-практ. конф. Северо-Западный институт управления РАНХиГС при Президенте РФ. СПб.: Изд-во ИИУНЦ «Стратегия будущего», 2017. С. 45–49.

ЧЕРНОГОРСКИЙ Сергей Александрович. E-mail: chernog_sa@spbstu.ru

ШВЕЦОВ Константин Владимирович. E-mail: shvetsov@inbox.ru

ХОДЫРЕВ Владимир Владимирович. E-mail: Khodyreffvladimir@yandex.ru

Статья поступила в редакцию 14.09.17

REFERENCES

- [1] **K.M. Murphy, A. Shleifer, R.W. Vishny,** Why is Rent-Seeking So Costly to Growth? The American Economic Review, 83 (2) (1993) 409–414.
- [2] **M. Levine, G. Satarov,** Rent-oriented Russia, Voprosy ekonomiki, 01 (2014) 61–77.
- [3] **R. Feenstra, A. Taylor,** International Economics. Worth Publishers, New York, 2008. ISBN 978-0-7167-9283-3
- [4] **A. Krueger,** The Political Economy of the Rent-Seeking Society, American Economic Review, 64 (3) (1974) 291–303.
- [5] **G. Tullock,** The Welfare Costs of Tariffs, Monopolies, and Theft, Western Economic Journal, 5 (3) (1967) 224–232.
- [6] **J. Buchanan, G. Tullock, R. Tollison,** Toward a Theory of the Rent-Seeking Society, Texas, Texas A&M University, Texas a & M University Economics Series, 1980.
- [7] **G. Tullock,** Efficient rent-seeking, Buchanan J.M., Tullock G. Tollison R.D. Toward a Theory of the Rent-Seeking Society, Texas A&M. University Press, 1980.

- [8] **W.J. Corcoran**, Long-run equilibrium and total expenditures in rent-seeking Public Choice, 43 (1) (1984) 89–94.
- [9] **P.T. Leeson**, The Invisible Hook: The Hidden Economics of Pirates, Princeton University Press, (2009) 191.
- [10] **V.P. Busygin, E.V. Zhelobod'ko, A.A. Tsyplakov**, Microeconomics – third level, Novosibirsk, SB RAS, 2003.
- [11] **J.J.J. Thijssen, K.J.M. Huisman, P.M. Kort**, (2012) Symmetric equilibrium strategies in game theoretic real option models Journal of Mathematical Economics, 48 (4) (2012) 219–225.
- [12] **S. Nitzan**, Collective Rent Dissipation, Economic Journal, 101 (402) (1991) 1522–1534.
- [13] **J. Mokyr, J.V.C. Nye**, Distributional Coalitions, the Industrial Revolution, and the Origins of Economic Growth in Britain, Southern Economic Journal, 74 (1) (2007) 50–70.
- [14] **K. Baik, S. Lee**, Collective Rent Seeking with Endogenous Group Sizes, European Journal of Political Economy, 13 (1) (1997) 121–130.
- [15] **H. Cavusoglu, S. Raghunathan, W.T. Yue**, Decision-Theoretic and game-theoretic approaches to IT security investment, Journal of Management Information Systems, 25 (2) (2008) 281–304.
- [16] **S.A. Chernogorskiy**, Rent-Seeking Behavior of Russian and Foreign Car Manufacturers, Integration in economy of the world economic system: Collection of scientific works of the 10th International Conference, St. Petersburg, Publishing house of St. Petersburg state technical university, (2005) 181–185.
- [17] **E.C. Pasour**, Rent Seeking: Some Conceptual Problems and Implications, The Review of Austrian Economics, 1 (1987) 123–145.
- [18] **A.E. Shastitko**, Institutional economic theory, Moscow, Theis publishing house, 1997.
- [19] **R. Schenk**, «Rent Seeking». CyberEconomics. Archived from the original on January 3, 2006. Retrieved 2007-02-11.
- [20] **V.D. Matveyenko, A.V. Korolev, M.A. Bakhtin**, Igrovyye ravnovesiya v setevykh modelyakh s raznoy stepenyu zavisimosti ot sredy, Gosudarstvo i biznes. Sovremennyye problemy ekonomiki: Materialy IX Mezhdunarodnoy nauchno-prakticheskoy konferentsii, Severo-Zapadnyy institut upravleniya RANKiGS pri Prezidente RF, St. Petersburg, Strategiya budushchego, (2017) 45–49.

CHERNOGORSKIY Sergej A. E-mail: chernog_sa@spbstu.ru
SHVETSOV Konstantin V. E-mail: shvetsov@inbox.ru
KHODYREV Vladimir V. E-mail: Khodyreffvladimir@yandex.ru