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THE ELABORATION OF AN ECONOMIC AND MATHEMATHICAL MODELS FOR THE PRODUCTION PROGRAM OF THE ENTERPRISE

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МОДЕЛИРОВАНИЕ ПРОИЗВОДСТВЕННОЙ ПРОГРАММЫ ПРЕДПРИЯТИЯ

Economic and mathematical models for the enterprise production program with homogenous product have been elaborated. Account passing product stocks, free store capacity, average labor in the product, market demand, and other basic resources have been taken into account. These models are used as the base for analogical models with heterogeneous product, allowing optimally coordinate material flows and enterprise's financial resources in a given period.

ECONOMIC AND MATHEMATICAL MODEL. PRODUCTION PROGRAM. ACTIVITY. RESOURCES. FLOWS. PRODUCT. ENTERPRISE.

Разработаны экономико-математические модели формирования производственной программы предприятия при однопродуктовой деятельности с учетом переходящих запасов готовой продукции, свободной емкости склада, трудоемкости изготовления продукции, рыночного спроса и других основных ресурсов. На их базе сформированы модели данной задачи при многопродуктовой деятельности, позволяющие оптимально согласовывать материальные потоки и имеющиеся финансовые ресурсы предприятия в рассматриваемом плановом периоде.

ЭКОНОМИКО-МАТЕМАТИЧЕСКАЯ МОДЕЛЬ. ПРОИЗВОДСТВЕННАЯ ПРОГРАММА. ДЕЯТЕЛЬНОСТЬ. РЕСУРСЫ. ПОТОКИ. ПРОДУКЦИЯ. ПРЕДПРИЯТИЕ.

Reducing costs of manufacturing and selling product and increasing business effectiveness results from a successful transformation in a business organization and technologies. At the same time, the plausibility and effectiveness of such a transformation increases, if its character matches to a large extent the nature of the relevant and effective processes that constitute the enterprise's development strategy.

Let's consider this transformation in a system of interaction between production and commercial functions of the enterprise. It includes a complex of organizational and economic measures to improve the system of interaction of production and commercial functions of the enterprise, including the distribution channels and nets system.

For the given enterprise, according to its specificity, let's use a modification with a one-level distribution channel – the manufacturing of one product, and a store with several sales departments.

1. Models of the production program for the business with one homogenous product

Costs of production, storage and sales of a product are determined and storage of the production is vary in different periods. The business is carried out in the given T time periods, this period is divided into separate equal time periods ($t = \overline{1, T}$) [1, 6, 7, 9].

The price is determined and can be vary in different time periods. Storage occurs in a limited capacity stock. The business can be carried out at its own expense or by credit.

Modeling the business with homogenous product at own expense

Given the assumed premises, the production and sales plan can be represented as a linear optimizing task.

Let's denote the unknown variables as: x_t – volume of production in t time period ($x_t \in N$, $t = \overline{1, T}$); y_{tj} – volume of sales in t time period through the sales department j ($y_{tj} \in N$, $t = \overline{1, T}$, $j = \overline{1, n}$).

Other parameters are: A – initial stock of the product; B – allowable storage capacity; A_{T+1} – rolling stock of the product in $T + 1$ time period; c_t – costs of manufacturing and storing one item of the product in t time period; c'_{tj} – costs of selling one item of the product in t time period through the sales department j; C'_t – allocations of equity in t time period; $в_{ut}$ – labor-output ratio of manufacturing one item of the product using equipment group u in t time period; $Ф_{эфut}$ – effective working time of equipment group u in t time period; $d_{\ell t}$ – amount of ℓ resource for one item of the product in t time period; $D'_{\ell t}$ – allocations of ℓ resource in t time period; M_{tj}, M'_{tj} – respectively lower and upper limits of sales in t time period through the sales department j; p_t – product price in t time period.

Thus, the model of the business with a homogenous product at own expense has the following form:

The volume of production and initial stock in r time periods cannot be less than the volume of sales in these r time periods:

$$\sum_{t=1}^{r} y_t - \sum_{t=1}^{r} x_t \le A, \quad h = \overline{1,H}; \quad r = \overline{1, T-1}; \tag{1}$$

$$\sum_{t=1}^{T} x_t - \sum_{t=1}^{T} y_t = A_{T+1} - A, \quad r = T. \tag{2}$$

The volume of production in r time periods cannot be more than the volume of sales and free storage capacity in these r time periods:

$$\sum_{t=1}^{r} x_t - \sum_{t=1}^{r} y_t \le B - A, \quad r = \overline{1, T}. \tag{3}$$

The total costs in r time periods cannot be more than allocations of equity in these r time periods:

$$\sum_{t=1}^{r} c_t x_t \le \sum_{t=1}^{r} C'_t, \quad r = \overline{1, T}. \tag{4}$$

The total labor-output ratio in r time periods cannot be more than effective working time of equipment groups in these r time periods:

$$\sum_{t=1}^{r} в_{ut} x_t \le \sum_{t=1}^{r} Ф_{эфut}, \quad u = \overline{1, U}; \quad r = \overline{1, T}. \tag{5}$$

The total use of other resources in r time periods cannot be more than allocations of resources in these r time periods:

$$\sum_{t=1}^{r} d_{\ell t} x_t \le \sum_{t=1}^{r} D'_{\ell t}, \quad \ell = \overline{1, L}, \quad r = \overline{1, T}. \tag{6}$$

Restrictions of sales, determined by demand:

$$M_{tj} \le y_{tj} \le M'_{tj}, \quad t = \overline{1, T}; \quad j = \overline{1, n}. \tag{7}$$

The maximum profit in T time periods is:

$$f(x, y) = \sum_{t=1}^{r} p_t\, y_t - \sum_{t=1}^{T} c_t x_t \to \max. \tag{8}$$

Modeling the business with one homogenous product by credit

In forming production program by simple interest credit, got in the start and fully repaid in the finish of the plan time period T, in the model (1)–(8), instead of the restriction (4), the restriction (9) is introduced, and the objective function (8) is changed [2, 8].

The credit and volume of sales in r time periods cannot be less than costs and payments for the credit in these r time periods:

$$\kappa + \sum_{t=1}^{r} p_t y_t \ge \sum_{t=1}^{r} c_t x_t + k \sum_{t=1}^{r} \varepsilon_t, \quad r = \overline{1, T}; \tag{9}$$

$$f(x, y, \kappa) = \sum_{t=1}^{T} p_t y_t - \sum_{t=1}^{T} c_t x_t - \left[1 + \sum_{t=1}^{T} \varepsilon_t\right] \kappa \to \max, \tag{10}$$

where κ – unknown volume of the starting credit ($\kappa \ge 0$); ε_t – interest rate in t ($t = \overline{1, T}$) time period.

If repayment of the credit is carried out in stages in t plan time periods, than the restriction (9) and the objective function (10) are changed as follows:

$$\sum_{t=1}^{r} c_t x_t - \sum_{t=1}^{r} \sum_{j=1}^{n} \left(p_t - c'_{tj}\right) y_{tj} \le \left(1 - \sum_{t=1}^{r-1} E_t - \sum_{t=1}^{r} \left(1 - \sum_{t=1}^{r-1} E_t\right) \varepsilon_t\right) \kappa, \quad r = \overline{1, T}; \tag{11}$$

$$f(x, y, \hat{e}) = \sum_{t=1}^{\mathrm{T}} p_t y_t - \sum_{t=1}^{T} \tilde{n}_t x_t - \left[1 + \sum_{t=1}^{T}\left(1 - \sum_{t=1}^{T-1} E_t\right)\varepsilon_t\right]\hat{e} \to \max, \quad (12)$$

where E_t – a part of the credit repayment in t time period on the condition that $\sum_{t=1}^{T} E_t = 1$.

With compound interest lending, getting the credit in the start and full repayment in the finish of the plan time period T, the restriction (9) and the objective function (10) are changed respectively:

$$\sum_{t=1}^{\mathrm{r}} \tilde{n}_t x_t - \sum_{t=1}^{r}\sum_{j=1}^{n} p_t y_t \le \le \left(1 - \sum_{t=1}^{r}\left[(1+\varepsilon)^t - 1\right]\right)\hat{e}, \quad r = \overline{1, T}; \quad (13)$$

$$f(x, y, \hat{e}) = \sum_{t=1}^{T} p_t y_t - \sum_{t=1}^{T} \tilde{n}_t x_t - \left[1 + \sum_{t=1}^{T}\left[(1+\varepsilon)^t - 1\right]\right]\hat{e} \to \max. \quad (14)$$

If repayment of the compound interest credit is carried out in stages in t plan time periods, than the restriction (11) and the objective function (12) have the following form:

$$\sum_{t=1}^{\mathrm{r}} c_t x_t - \sum_{t=1}^{\mathrm{r}} p_t y_t \le \le \left\{1 - \sum_{t=1}^{r-1} E_t - \sum_{t=1}^{r}\left(1 - \sum_{t=1}^{r-1} E_t\right)\left[(1+\varepsilon)^t - 1\right]\right\}\kappa, \quad r = \overline{1, T}; \quad (15)$$

$$f(x, y, \kappa) = \sum_{t=1}^{\mathrm{T}} p_t y_t - \sum_{t=1}^{\mathrm{T}} c_t x_t - \left\{1 + \sum_{t=1}^{r}\left(1 - \sum_{t=1}^{r-1} E_t\right)\left[(1+\varepsilon)^t - 1\right]\right\}\kappa \to \max. \quad (16)$$

2. Models of the production program for the business with heterogeneous product

Models of the production program at own expense

The model of forming the production program at own expense has the following form: [11]

$$\sum_{t=1}^{r} y_{ht} - \sum_{t=1}^{r} x_{ht} \le A_h, \quad h = \overline{1, H}; \quad r = \overline{1, T-1}; \quad (17)$$

$$\sum_{t=1}^{T} x_{ht} - \sum_{t=1}^{T} y_{ht} = A_{hT+1} - A_h, \quad h = \overline{1, H}; \quad r = T; \quad (18)$$

$$\sum_{t=1}^{r}\sum_{h=1}^{H} x_{ht} v_h - \sum_{t=1}^{r}\sum_{h=1}^{H} y_{ht} v_h \le \le B - \sum_{h=1}^{H} A_h v_h, \quad r = \overline{1, T}; \quad (19)$$

$$\sum_{t\ 1}^{r}\sum_{h=1}^{H} c_{ht} x_{ht} \le \sum_{t=1}^{r} C'_t, \quad r = \overline{1, T}; \quad (20)$$

$$\sum_{t=1}^{r}\sum_{h=1}^{H} в_{hut} x_{ht} \le \sum_{t=1}^{r} \Phi_{эфut}, \quad u = \overline{1, U}; \quad r = \overline{1, T}; \quad (21)$$

$$\sum_{t=1}^{r}\sum_{h=1}^{H} d_{h\ell t} x_{ht} \le \sum_{t=1}^{r} D'_{\ell t}, \quad \ell = \overline{1, L}; \quad r = \overline{1, T}; \quad (22)$$

$$M_{htj} \le y_{htj} \le M'_{htj}, \quad j = \overline{1, n}; \quad h = \overline{1, H}; \quad t = \overline{1, T}; \quad (23)$$

$$f(x, y) = \sum_{t=1}^{T}\sum_{h=1}^{H} p_{ht} y_{ht} - \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht} x_{ht} \to \max. \quad (24)$$

In this model, all legend is identical to the model with one homogenous product excluding the additional index h $(h = \overline{1, H})$ of h product, and v_h – volume of this product.

Models of the production program by credit

In modeling business with heterogeneous product by simple interest credit, got in the start and fully repaid in the finish of the plan time period T, in the model (17)–(24), instead of the restriction (20), the restriction (25) is introduced, and the objective function (24) is changed [7, 8].

$$\sum_{t=1}^{r}\sum_{h=1}^{H} c_{ht} x_{ht} - \sum_{t=1}^{r}\sum_{h=1}^{H} p_{ht} y_{ht} \le \le \left(1 - \sum_{t=1}^{r} \varepsilon_t\right)\kappa, \quad r = \overline{1, T}; \quad (25)$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{h=1}^{H} p_{ht} y_{ht} - \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht} x_{ht} - \left[1 + \sum_{t=1}^{T} \varepsilon_t\right]\kappa \rightarrow \max. \quad (26)$$

If repayment of the credit is carried out in stages in t plan time periods, than the restriction (25) and the objective function (26) are changed respectively:

$$\sum_{t=1}^{r}\sum_{h=1}^{H} c_{ht} x_{ht} - \sum_{t=1}^{r}\sum_{h=1}^{H} p_{ht} y_{ht} \le \left(1 - \sum_{t=1}^{r-1} E_t - \sum_{t=1}^{r}\left(1 - \sum_{t=1}^{r-1} E_t\right)\varepsilon_t\right)\kappa, \quad r = \overline{1, T}; \quad (27)$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{h=1}^{H} p_{ht} y_{ht} - \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht} x_{ht} - \left[1 + \sum_{t=1}^{T}\left(1 - \sum_{t=1}^{T-1} E_t\right)\varepsilon_t\right]\kappa \rightarrow \max. \quad (28)$$

In modeling business with heterogeneous product by compound interest credit, got in the start and fully repaid in the finish of the plan time period T, in the model (17)–(24), instead of the restriction (20), the restriction (29) is introduced, and the objective function (24) is changed:

$$\sum_{t=1}^{r}\sum_{h=1}^{H} c_{ht} x_{ht} - \sum_{t=1}^{r}\sum_{h=1}^{H} p_{ht} y_{ht} \le \left(1 - \sum_{t=1}^{r} (1+\varepsilon)^t - 1\right)\kappa, \quad r = \overline{1, T}; \quad (29)$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{h=1}^{H} p_{ht} y_{ht} - \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht} x_{ht} - \left[1 + \sum_{t=1}^{T} (1+\varepsilon)^t - 1\right]\kappa \rightarrow \max. \quad (30)$$

If repayment of the compound interest credit is carried out in stages in t plan time periods, than the restriction (27) and the objective function (28) have the following form:

$$\sum_{t=1}^{r}\sum_{h=1}^{H} c_{ht} x_{ht} - \sum_{t=1}^{r}\sum_{h=1}^{H} p_{ht} y_{ht} \le \left\{1 - \sum_{t=1}^{r-1} E_t - \sum_{t=1}^{r}\left(1 - \sum_{t=1}^{r-1} E_t\right)\left[(1+\varepsilon)^t - 1\right]\right\}\kappa, \quad (31)$$

$$r = \overline{1, T};$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{h=1}^{H} p_{ht} y_{ht} - \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht} x_{ht} - \left[1 + \sum_{t=1}^{T}\left(1 - \sum_{t=1}^{T-1} E_t\right)\left[(1+\varepsilon)^t - 1\right]\right]\kappa \rightarrow \max. \quad (32)$$

The formed tasks are linear optimizing tasks.

With the distribution channels (shops or sales departments) described models are modified this way.

3. Modification of the formed models with distribution channels

Modeling the business with one homogenous product at own expense

Let's denote the unknown variables as: x_t – volume of production in t time period ($x_t \in N$, $t = \overline{1, T}$); y_{tj} – volume of sales in t time period though the sales department j ($y_{tj} \in N$, $t = \overline{1, T}$, $j = \overline{1, n}$) [4, 5, 10].

Other parameters are: A – initial stock of the product; B – allowable storage capacity; A_{T+1} – rolling stock of the product in $T+1$ time period; c_t – costs of manufacturing and storing one item of the product in t time period; c'_{tj} – costs of selling one item of the product in t time period though the sales department j; C'_t – allocations of equity in t time period; $в_{ut}$ – labor-output ratio of manufacturing one item of the product using equipment group u in t time period; $Ф_{эфut}$ – effective working time of equipment group u in t time period; $d_{\ell t}$ – use of ℓ resource on one item of the product in t time period; $D'_{\ell t}$ – allocations of ℓ resource in t time period; M_{tj}, M'_{tj} – respectively lower and upper limits of sales in t time period though the sales department j; p_t – product price in t time period.

Thus, the model of the business with one homogenous product at own expense has the following form:

The volume of production and initial stock in r time periods cannot be less than the volume of sales in these r time periods:

$$\sum_{t=1}^{r}\sum_{j=1}^{n} y_{tj} - \sum_{t=1}^{r} x_t \le A, \quad r = \overline{1, T-1}; \quad (33)$$

$$\sum_{t=1}^{T} x_t - \sum_{t=1}^{T}\sum_{j=1}^{n} y_{tj} = A_{T+1} - A, \quad r = T. \quad (34)$$

The volume of production in r time periods cannot be more than the volume of sales and free storage capacity in these r time periods:

$$\sum_{t=1}^{r} x_t - \sum_{t=1}^{r}\sum_{j=1}^{n} y_{tj} \le B - A, \quad r = \overline{1, T}. \quad (35)$$

The total costs in r time periods cannot be more than allocations of equity in these r time periods:

$$\sum_{t=1}^{r} c_t x_t \le \sum_{t=1}^{r} C_t', \quad r = \overline{1, T}. \quad (36)$$

The total labor-output ratio in r time periods cannot be more than effective working time of equipment groups in these r time periods:

$$\sum_{t=1}^{r} в_{ut} x_t \le \sum_{t=1}^{r} \Phi_{эфut}, \quad u = \overline{1, U}; \quad r = \overline{1, T}. \quad (37)$$

The total use of other resources in r time periods cannot be more than allocations of these resources in these r time periods:

$$\sum_{t=1}^{r} d_{\ell t} x_t \le \sum_{t=1}^{r} D_{\ell t}', \quad \ell = \overline{1, L}, \quad r = \overline{1, T}. \quad (38)$$

Restrictions of sales, determined by demand:

$$M_{tj} \le y_{tj} \le M_{tj}', \quad t = \overline{1, T}; \quad j = \overline{1, n}. \quad (39)$$

The maximum profit in T time periods is:

$$f(x, y) = \sum_{t=1}^{r}\sum_{j=1}^{n} (p_t - c_{tj}') y_{tj} - \sum_{t=1}^{T} c_t x_t \to \max. \quad (40)$$

Models of forming production program by credit have the following form:

In forming production program by simple interest credit, got in the start and fully repaid in the finish of the plan time period T, in the model (33)–(40), instead of the restriction (36), the restriction (41) is introduced, and the objective function (40) is changed:

The credit and volume of sales in r time periods cannot be less than costs and payments for the credit in these r time periods:

$$\kappa + \sum_{t=1}^{r}\sum_{j=1}^{n} (p_t - c_{tj}') y_{tj} \ge \sum_{t=1}^{r} c_t x_t + \sum_{t=1}^{r} \varepsilon_t \kappa, \quad r = \overline{1, T}; \quad (41)$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{j=1}^{n} (p_t - c_{tj}') y_{tj} - \sum_{t=1}^{T} c_t x_t - \left[1 + \sum_{t=1}^{T} \varepsilon_t\right]\kappa \to \max, \quad (42)$$

where κ – unknown volume of the starting credit ($\kappa \ge 0$); ε_t – interest rate in t ($t = \overline{1, T}$) time period.

If repayment of the credit is carried out in stages in t plan time periods, than the restriction (41) and the objective function (42) are changed as follows:

$$\sum_{t=1}^{r} c_t x_t - \sum_{t=1}^{r}\sum_{j=1}^{n} (p_t - c_{tj}') y_{tj} \le \left(1 - \sum_{t=1}^{r-1} E_t - \sum_{t=1}^{r}\left(1 - \sum_{t=1}^{r-1} E_t\right)\varepsilon_t\right)\kappa, \quad r = \overline{1, T}; \quad (43)$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{j=1}^{n} (p_t - c_{tj}') y_{tj} - \sum_{t=1}^{T} c_t x_t - \left[1 + \sum_{t=1}^{T}\left(1 - \sum_{t=1}^{T-1} E_t\right)\varepsilon_t\right]\kappa \to \max, \quad (44)$$

where E_t – a part of the credit repayment in t time period on the condition that $\sum_{t=1}^{T} E_t = 1$.

With compound interest lending, getting the credit in the start and full repayment in the finish of the plan time period T, the restriction (43) and the objective function (44) are changed respectively:

$$\sum_{t=1}^{r} c_t x_t - \sum_{t=1}^{r}\sum_{j=1}^{n} (p_t - c_{tj}') y_{tj} \le \left(1 - \sum_{t=1}^{r}\left[(1+\varepsilon)^t - 1\right]\right)\kappa, \quad r = \overline{1, T}; \quad (45)$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{j=1}^{n} (p_t - c_{tj}') y_{tj} - \sum_{t=1}^{T} c_t x_t - \left[1 + \sum_{t=1}^{T}\left[(1+\varepsilon)^t - 1\right]\right]\kappa \to \max. \quad (46)$$

If repayment of the compound interest credit is carried out in stages in t plan time periods, than the restriction (45) and the objective function (46) have the following form:

$$\sum_{t=1}^{r} c_t x_t - \sum_{t=1}^{r}\sum_{j=1}^{n} (p_t - c'_{tj}) y_{tj} \le$$
$$\le \left\{1 - \sum_{t=1}^{r-1} E_t - \sum_{t=1}^{r}\left(1 - \sum_{t=1}^{r-1} E_t\right)\left[(1+\varepsilon)^t - 1\right]\right\}\kappa, \quad (47)$$
$$r = \overline{1, T};$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{j=1}^{n} (p_t - c'_{tj}) y_{tj} - \sum_{t=1}^{T} c_t x_t -$$
$$- \left\{1 + \sum_{t=1}^{r}\left(1 - \sum_{t=1}^{r-1} E_t\right)\left[(1+\varepsilon)^t - 1\right]\right\}\kappa \to \max. \quad (48)$$

Models of the production program for the business with heterogeneous product

The model of forming the production program at own expense has the following form [6]:

$$\sum_{t=1}^{r}\sum_{j=1}^{n} y_{htj} - \sum_{t=1}^{r} x_{ht} \le A_h, \quad (49)$$
$$h = \overline{1, H}; \quad r = \overline{1, T-1};$$

$$\sum_{t=1}^{T} x_{ht} - \sum_{t=1}^{T}\sum_{j=1}^{n} y_{htj} = A_{hT+1} - A_h, \quad (50)$$
$$h = \overline{1, H}; \quad r = T;$$

$$\sum_{t=1}^{r}\sum_{h=1}^{H} x_{ht} v_h - \sum_{t=1}^{r}\sum_{h=1}^{H} y_{htj} v_h \le$$
$$\le B - \sum_{h=1}^{H} A_h v_h, \quad r = \overline{1, T}; \quad (51)$$

$$\sum_{t=1}^{r}\sum_{h=1}^{H} c_{ht} x_{ht} \le \sum_{t=1}^{r} C'_t, \quad r = \overline{1, T}; \quad (52)$$

$$\sum_{t=1}^{r}\sum_{h=1}^{H} в_{hut} \le \sum_{t=1}^{r} Ф_{эфut}, \quad (53)$$
$$u = \overline{1, U}; \quad r = \overline{1, T};$$

$$\sum_{t=1}^{r}\sum_{h=1}^{H} d_{h\ell t} x_{ht} \le \sum_{t=1}^{r} D'_{\ell t}, \quad (54)$$
$$\ell = \overline{1, L}; \quad r = \overline{1, T};$$

$$M_{htj} \le y_{htj} \le M'_{htj}, \quad j = \overline{1, n}; \quad (55)$$
$$h = \overline{1, H}; \quad t = \overline{1, T};$$

$$f(x, y) = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{j=1}^{n} \left(p_{ht} - c'_{htj}\right) y_{htj} -$$
$$- \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht} x_{ht} \to \max. \quad (56)$$

In this model, all legend is identical to the model with one homogenous product excluding the additional index j $(j = \overline{1, n})$ of j sales department, and c'_{tj} – costs of selling one item of the product in t time period though the sales department *j*.

Models of the production program by credit

In modeling business with heterogeneous product by simple interest credit, got in the start and fully repaid in the finish of the plan time period *T*, in the model (49)–(56), instead of the restriction (52), the restriction (57) is introduced, and the objective function (56) is changed:

$$\sum_{t=1}^{r}\sum_{h=1}^{H} c_{ht} x_{ht} - \sum_{t=1}^{r}\sum_{h=1}^{H}\sum_{j=1}^{n} \left(p_{ht} - c'_{htj}\right) y_{htj} \le$$
$$\le \left(1 - \sum_{t=1}^{r} \varepsilon_t\right)\kappa, \quad r = \overline{1, T}; \quad (57)$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{j=1}^{n} \left(p_{ht} - c'_{htj}\right) y_{htj} -$$
$$- \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht} x_{ht} - \left[1 + \sum_{t=1}^{T} \varepsilon_t\right]\kappa \to \max. \quad (58)$$

If repayment of the credit is carried out in stages in *t* plan time periods, than the restriction (57) and the objective function (58) are changed respectively:

$$\sum_{t=1}^{r}\sum_{h=1}^{H} c_{ht} x_{ht} - \sum_{t=1}^{r}\sum_{h=1}^{H}\sum_{j=1}^{n} \left(p_{ht} - c'_{htj}\right) y_{htj} \le$$
$$\le \left(1 - \sum_{t=1}^{r-1} E_t - \sum_{t=1}^{r}\left(1 - \sum_{t=1}^{r-1} E_t\right)\varepsilon_t\right)\kappa, \quad r = \overline{1, T}; \quad (59)$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{j=1}^{n} \left(p_{ht} - c'_{htj}\right) y_{htj} -$$
$$- \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht} x_{ht} - \quad (60)$$
$$- \left[1 + \sum_{t=1}^{T}\left(1 - \sum_{t=1}^{T-1} E_t\right)\varepsilon_t\right]\kappa \to \max.$$

In modeling business with heterogeneous product by compound interest credit, received in the beginning and fully repaid at the end of the plan time period T, in the model (49)–(56), instead of the restriction (52), the restriction (61) is introduced, and the objective function (56) is changed:

$$\sum_{t=1}^{r}\sum_{h=1}^{H} c_{ht}x_{ht} - \sum_{t=1}^{r}\sum_{h=1}^{H}\sum_{j=1}^{n}\left(p_{ht} - c'_{htj}\right)y_{htj} \le \le \left(1 - \sum_{t=1}^{r}(1+\varepsilon)^{t} - 1\right)\kappa, \quad r = \overline{1, T}; \tag{61}$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{j=1}^{n}\left(p_{ht} - c'_{htj}\right)y_{htj} - - \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht}x_{ht} - \left[1 + \sum_{t=1}^{T}(1+\varepsilon)^{t} - 1\right]\kappa \to \max. \tag{62}$$

If repayment of the compound interest credit is carried out in stages in t plan time periods, the restriction (59) and the objective function (60) have the following form:

$$\sum_{t=1}^{r}\sum_{h=1}^{H} c_{ht}x_{ht} - \sum_{t=1}^{r}\sum_{h=1}^{H}\sum_{j=1}^{n}\left(p_{ht} - c'_{htj}\right)y_{htj} \le \le \left\{1 - \sum_{t=1}^{r-1} E_t - \sum_{t=1}^{r}\left(1 - \sum_{t=1}^{r-1} E_t\right)\left[(1+\varepsilon)^{t} - 1\right]\right\}\kappa, \quad r = \overline{1, T} \tag{63}$$

$$f(x, y, \kappa) = \sum_{t=1}^{T}\sum_{h=1}^{H}\sum_{j=1}^{n}\left(p_{ht} - c'_{htj}\right)y_{htj} - - \sum_{t=1}^{T}\sum_{h=1}^{H} c_{ht}x_{ht} - \left[1 + \sum_{t=1}^{T}\left(1 - \sum_{t=1}^{T-1} E_t\right)\times \times \left[(1+\varepsilon)^{t} - 1\right]\right]\kappa \to \max. \tag{64}$$

The formed tasks are linear optimizing tasks.

Thus, the elaborated economic and mathematical models for the production program of the enterprise with one homogenous product with taking into account rolling product stocks, free storage capacity, product labor-output ratio, demand and other resources both at own expense and by simple and (or) compound interest credit differ by forming the set of trajectories making the tasks by calculating the pessimistic and optimistic variants of its solution in terms of the objective function – profit maximum. Based on the elaborated models with one homogenous product, economic and mathematical models of determining production program of the enterprise with heterogeneous product have been formed. Account conjoint usage free storage capacity, effective working time of equipment groups, financial and other resources needed for making the heterogeneous product at own expense and (or) by credit have been taken into account.

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