UDK 330.322.214

L.V. Nikolova, Ju.V. Radionova

A MODEL FOR THE OPTIMIZATION OF A REGIONAL INVESTMENT PROGRAM

Л.В. Николова, Ю.В. Радионова

МОДЕЛЬ ОПТИМИЗАЦИИ ИНВЕСТИЦИОННОЙ ПРОГРАММЫ РЕГИОНА

The article reviews the question of building a regional investment program in the context of limited resources. When building a regional investment program in the context of limited resources, a need arises to construct a portfolio of orders with regard to their hierarchical priority, which, in this case, means a criterion for optimization. Using this approach, a program includes, first of all, objects which have qualitatively assessable indicators with higher values. To solve this task, we can use mathematical modeling tools and optimization (normative) tools containing expressions with algebraic operations, which can be maximized or minimized with certain limitations.

INVESTMENT PROGRAM. OPTIMIZATION. PRIORITY. REGION. HIERARCHICAL ANALYSIS METHOD.

Рассмотрен вопрос формирования инвестиционной программы региона в условиях ограниченности ресурсов. При формировании инвестиционной программы региона в условиях ограниченности ресурсов возникает необходимость формирования «портфеля заказов» с учётом их иерархической приоритетности, что является в данном случае критерием оптимизации. При таком подходе в программу включают, в первую очередь, объекты, имеющие количественно измеренные более высокие показатели. Для решения этой задачи можно использовать аппарат математического моделирования, модели оптимизации (нормативные), содержащие выражения с алгебраическими операциями, которые можно максимизировать или минимизировать при определённых ограничениях.

ИНВЕСТИЦИОННАЯ ПРОГРАММА. ОПТИМИЗАЦИЯ.ПРИОРИТЕТНОСТЬ. РЕГИОН. МЕТОД АНАЛИЗА ИЕРАРХИЙ.

Introduction. At the current stage of market relations development, unlike in preceding periods, a priority assessment of permanent facilities is a must for the purposes of the development and optimization of a regional capital construction program. To solve this task, we can use mathematical modeling tools and optimization (normative) tools containing expressions with algebraic operations, which can be maximized or minimized with certain limitations. Priority assessment criteria should conform to the following requirements:

goals and objectives of investment activity participants must be taken into account as fully as possible;

possibilities for constructing and developing investment objects, along with investment outcomes, must be assessed comprehensively with due regard for technical and economic characteristics of capital construction;

an ordered hierarchical set must be applied in its composition and content [1].

This set of data must underlie the process of building a hierarchy of criteria, which, in turn, is a structural organization of compound multilayered systems.

1. A mathematical economic model of priority assessment for capital construction objects

Let us build a mathematical economic model of priority assessment for objects of capital construction. For this purpose, application of the method of paired comparison seems to be most practical; the method is used when compared objects can be matched only subjectively, i. e. when precise measurements are impossible to help decide which of the two is more preferable. The chief goal of this comparison is to organize objects. The Tab. 1 shows a set of criteria for object priority assessment to be used for the

Table 1

Criteria for priority assessment of capital construction objects

Groups of criteria	The content of a group of criteria
Target criteria	Composition of target criteria is defined by performance/operational requirements of investments object users. Overlapping of some target criteria with criteria of other groups is acceptable.
External and environmental criteria	Object construction enforceability
	Influence of an investments object on employment rates
	Environmental friendliness of object construction
Criteria by EPCM carrying out the construction works	Brief description of EPCM's management personnel
	Financial stability of the EPCM
	EPCM's economic activity results and tendencies
Engineering criteria	Maturity level of the construction object
	Technical and economic parameters of the object
	Positive impact on other objects
Economic criteria	Construction costs
	Risks related to the allocation of capital investment for the construction
Regional-specific criteria for object construction	Object construction compliance with regional legislation
	Regional resource potential
	Regional infrastructure situation (communications and banking services)

development of regional investment programs. When using the paired comparison method, we need a qualitative assessment to help us objectively assess the compared pairs of elements with the aim of revealing the preferred ones [2].

There are three main forms to obtain data on the preferability of this or that element:

– quantitative data based on substitutions;

– information based on the interpretations of a linguistic variable: to compare alternatives, meanings of a linguistic variable are specified – «approximately equivalent», «slightly better», «significantly better», etc.;

– information in the form of an ordinal scale.

The application of the most convenient ordinal scale for paired comparisons in the range between 0 and ∞ may prove useless because human faculty of discerning is limited. Therefore the scale must be a limited range, in accordance with the possibility of making relational assessments. Because 1 is a standard unity for measurements [3], the upper limit of the scale must not be too far from this value.

On the other hand, this range must at the same time correspond to the ability of a decision-maker to perceive changes in the value being measured. So, we have to increase the measured quantity s by a minimal value Δs, in order to reach the state when our perception is already capable of discerning between s and $s+\Delta s$.

Perception changes are observed when the measured quantity is increased by a constant percentage, which happens when Δs is insignificant if compared with s; perception change practically vanishes when s is too small or too large. If s_0 is the first value of the measured variable, then the next perceived change of it (s_1) will be defined in the following way:

$$s_1 = s_0 + \Delta s_0 = s_0 + \frac{\Delta s_0}{s_0} s_0 = s_0(1 + r). \quad (1)$$

Similarly,

$$s_2 = s_1 + \Delta s_1 = s_1(1 + r) = s_0(1 + r)^2 \equiv s_0\alpha^2. \quad (2)$$

In the general case,

$$s_n = s_{n-1}\alpha = s_0\alpha^n \text{ when } n = 0.\ 1,\ 2... \quad (3)$$

Thus, perceived changes of the variable s are ranged along the exponential progression. On the other hand, the time of perception is an arithmetic series of discrete points where only barely discernible differences are observed. These points can be defined if we solve equation (3) for n; as a result, we have:

$$n = \frac{\lg s_n - \lg s_0}{\lg \alpha}. \quad (4)$$

If we indicate perception by $M = \lg \acute{\alpha}$ and solve the equation (4) for this variable, we have:

$$M = \frac{1}{n} \lg s_n - \frac{\lg s_0}{n}. \quad (5)$$

By indicating $a = 1/n$ and $b = -\lg s_0$, we have the Weber-Fechner law:

$$M = a \lg s + b, \quad a \neq 0. \quad (6)$$

When perception is $M = 0$. which happens if the object (criterion) is compared to itself, it follows that if $b = 0$ $\lg s_0 = 0$ or $s_0 = 1$.

The next observable perception with $s_0 =$, according to equation (3) will be defined in the following way:

$$s_1 = s_0 \alpha = \alpha, \quad (7)$$

In formula (4), its value is $\lg \alpha / \lg \alpha = 1$. The next observable perception of the measured variable will be defined in the following way:

$$s_2 = s_0 \alpha^2. \quad (8)$$

It takes the value of «2». Thus we have a series of 1, 2, etc.

In practice, the key qualitative distinctions are few: approximately five of them, while additional ones are compromises between the adjacent key distinctions, thus making the total number = 9. Besides, there are other reasons for fixing the upper limit of the scale [4]:

– qualitative distinctions are significant, they have some accuracy when compared objects are homogeneous or close to each other in terms of the property (criterion) used for assessment;

– human faculty of making qualitative distinctions is very well represented by five definitions (equal, slight, strong, very strong, absolute), while, for even greater accuracy, we need to compromise definitions between the above five, which eventually gives nine meanings;

– for assessment of individual objects a *distinction zone trichotomy* is used (unacceptable, indifferent, and acceptable), and for their more precise classification this trichotomic principle underlies each of these zones: low degree, moderate degree, high degree, which, in its turn, produces another nine meanings.

Considering the above and bearing in mind that qualitative perceptions are subjective in nature and do change with time, we propose a relative significance scale [4] shown in Tab. 2 for object priority assessment.

This scale is applied in the following manner:

Table 2

Relative significance scale

Value of intensity r	Description of paired comparison situation
1	Equal significance of an element in a line and in a column.
2	A compromise between 1 and 3
3	The element in a string is slightly more superior in significance than the element in a column.
4	A compromise between 3 and 5
5	The element in a string is largely more superior in significance than the element in a column.
6	A compromise between 5 and 7
7	An element in a string is practically superior in significance than an element in a column.
8	A compromise between 7 and 9
9	An element in a string is obviously superior to an element in a column.
1/a	The above situations with a vice versa comparison of the same elements

In case $C_1, C_2, \ldots, C_n$ is a set of elements, quantitative expressions about pairs of elements (C_i, C_j) are presented on the relative significance scale as a matrix $n \times n$ of order $A = (a_{ij})$ with $ij = 1, 2, \ldots, n$. At the same time, matrix elements a_{ij} take the values of relative significance intensity r depending on the relevant correlation of criteria that are being compared.

When a criterion is compared with itself, 1 is written in the corresponding cell of matrix A

denoting equal value on the scale. When other pairs of criteria are compared, their values of relative significance intensity are defined depending on subjective assessments proceeding from the analysis of available input data.

Let us assume that criterion C has strong superiority over criterion B. Then in the matrix of paired comparisons, at the intersection of line C with column B «5» should be written (see Tab. 2), while at the intersection of string B with column C is the opposite value, i. e. «1/5», etc. This way, after all paired criteria comparisons have been done, the matrix or paired comparisons may take the following form:

$$A = \begin{bmatrix} 1 & a_{12} & \dots & a_{1n} \\ 1/a_{12} & 1 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 1/a_{1n} & 1/a_{2n} & \dots & 1 \end{bmatrix}. \quad (9)$$

For processing the resulting matrixes on the relative importance scale, we need an appropriate mathematical model [5]. At the same time, for reasonable model building we only need to reveal and consider all the main factors influencing the final result, and to reflect correlations between them with the sufficient entirety. Quantitative data obtained as a result of using the model will be used for developing a solution.

Let us denote a quantity corresponding to object significance x_i compared to x_j by a_{ij}. The matrix containing these numbers will be denoted as $A = (a_{ij})$, where $a_{ij} = 1/a_{ji}$. If comparisons (assessments) have been done in the right way, then $a_{ik} = a_{ij}a_{jk}$ for all i, j, k, and matrix A is called *consistent.* For such a matrix, there is an evident case when comparisons are based on precise measurement, i. e. when weights $w_1, \dots, w_n$ are known. Then $a_{ij} = w_i/w_j$ if $i, j = 1, \dots, n$ and, respectively, by inversion of indexes i and j in the expression a_{ij}, we obtain properties of the pared comparison matrix:

$$a_{ji} = w_j/w_i = \frac{1}{w_i/w_j} = \frac{1}{a_{ij}}. \quad (10)$$

In the matrix theory, a matrix equation $Ax = y$, where $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$ is equivalent to a short formula:

$$\sum_{j=1}^{n} a_{ij} w_j = n w_i, \; i = 1, \dots, n. \quad (11)$$

This is equivalent to an expression:

$$Aw = nw. \quad (12)$$

This formula reflects the fact that w is matrix A's proper vector with its own value n. Equation (12), if presented element by element, looks like this:

$$\begin{bmatrix} w_1/w_1 & w_1/w_2 & \dots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \dots & w_2/w_n \\ \dots & \dots & \dots & \dots \\ w_n/w_1 & w_n/w_2 & \dots & w_n/w_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix} = n \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix}. \quad (13)$$

Since a_{ij} is based not upon precise measurements but on subjective assessments, then a_{ij} will deviate from precise relations w_i / w_j. Therefore, equation (12) cannot be used in this form. Let us use two matrix properties:

1. If $\lambda_i, \dots, \lambda_n$ are numbers that satisfy equation $Ax = \lambda x$ and if $a_{ij} = 1$ for all i, then $\sum_{i=1}^{n} \lambda_i = n$.

2. Accordingly, if we have (12), then all its proper values = 0. except for one which is = n. In case of consistency, n is the largest proper value of A.

If elements a_{ij} of a positive matrix A are slightly changed, then proper values will also change insignificantly.

In this way, if a matrix diagonal consists of unities ($a_{ij} = 1$) and A is a consistent matrix, then with slight changes of a_{ij} the largest proper value λ_{max} will remain close to n, while other proper values will remain close to 0. Consequently, a mathematical economic model of priority assessment for objects of capital construction means developing results of paired comparisons of objects (criteria) into matrixes (9) and defining the matrixes' key proper vector satisfying the following condition:

$$Aw = \lambda_{max} w, \quad (14)$$

where A – matrix of values for object (criteria) paired assessments; w – key proper vector A, i. e. the vector of priority of compared objects (criteria); λ_{max} – the largest proper value of A.

Numerical values of the resulting vector $w = (w_1, \dots, w_n)^T$ are the priorities of corresponding elements that are being compared in the matrix.

To calculate priorities compared in the matrix of elements, let us use the following formula:

$$w_j = \frac{R_j}{\sum_{j=1}^{n} R_j}, \quad (15)$$

where $R_j = \left(\prod_{i=1}^{n} a_{ij} \right)^{1/n}$.

The calculation of the largest value of the main proper number of the matrix λ_{max} is done according to the following formula:

$$\lambda_{max} = \sum_{i=1}^{n} \sum_{j=1}^{n} w_j a_{ij}. \quad (16)$$

The resulting value λ_{max} is used to define consistency of paired comparisons in the model, which generally means that, provided we have the main array of unprocessed data, all other data can be obtained from them logically. To do paired comparisons of n objects, provided each of them is presented at least once, we will need $(n - 1)$ comparisons. We can deduce all other paired assessment from them using transitivity relation. The consistency within the considered economic mathematical model is equivalent to a requirement for equality of λ_{max} to the number of compared elements n. This way we can specify consistency deviance by determining the Consistency Relation (CR) and by its subsequent comparison with a threshold value:

$$ОС = ИС/СИ \le 0.2, \quad (17)$$

here $ИС = (\lambda_{max} - n) / (n - 1)$ – consistency index,

СИ – random index.

The random index is a consistency index of a matrix of the same dimension as A, which has been built randomly on the 1–9 scale but with correspondingly opposite values of its elements. The results of average random indexes' calculation for matrixes sized 1–15 are shown in Tab. 3.

Thus, the consistency relation imposes a limit upon the resulting economic mathematical model used to assess object priority. To control consistency and precision of object (criteria) priority values, their number must not exceed 15.

Table 3

Values of СИ

n	1	2	3	4	5	6	7	8
СИ	0	0	0.58	0.9	1.12	1.24	1.32	1.41

n	9	10	11	12	13	14	15
СИ	1.45	1.49	1.51	1.48	1.56	1.57	1.59

If the condition ОС < 0.2 is not met, causes for inconsistency are found and analyzed, and the correction of the paired comparisons of assessment criteria is done.

The sequence of actions aimed at defining local project priorities in relation to the criteria and checks of paired comparison consistency are the same as in the definition of local priorities of the assessment criteria. This means that the matrix of paired comparisons (9) must be built in the same way, but the comparison should be done not according to criteria importance, but according to the value of the criteria for individual objects. Then a proper vector is calculated for the matrix, whose (vector's) elements w_{ij} are determined using formula (15) and now reflect local priorities of the *i*-object of investment by *i*-criteria of comparison. After that, the matrix's proper number is calculated using formula (16) and a check is done for the fulfillment of the consistency condition using formula (17). If the consistency condition is not met, the correction of paired comparisons of criteria values is carried out.

After all arrays of stored data on local criteria priorities and investment projects related to these criteria have been calculated, a synthesis operation is carried out according to the following formula:

$$w_i^p = \sum_{j=1}^{n} w_j w_{ij}, \quad (18)$$

where w_i^p – integral priority of *i*-object; w_j – local priority of *j*-criterion; w_{ij} – local priority of *i*-object of investment by *j*-criterion; n – number of criteria.

The analysis has revealed that the regional investment object priority assessment has to be carried out using a set of criteria.

The obtained priority values help organize the objects according to their significance during the development and optimization of a regional capital construction program.

The economic mathematical basis of this model is the definition of the key proper vector if a paired comparison matrix has been built using the 9-point scale of relative significance. The numerical values of the resulting vector are priorities of the elements compared within the matrix. During this process, the consistency of paired comparisons is checked, which helps assess the calculation accuracy.

2. Application of the model to the development of an investment program

Let us look at possible applications of this economic mathematical model for determining a specific priority of construction objects exemplified by an investment program in one of North-Western regions of the Russian Federation.

To simplify the analysis, we assess only two objects planned for investment. To determine which of the two investment objects has a higher priority, we do a paired comparison of them. We choose the criteria according to a couple of principles: 1) the optimal use of regional resources already existent on the site of the planned construction; 2) availability of well-developed infrastructure.

Proceeding from these two principles, five criteria have been chosen:

1. Optimal use of construction equipment.
2. Available communications and utility lines.
3. Available electric power substation.
4. Development of infrastructure.
5. Environmental situation.

Having chosen the criteria, we do paired comparisons in order to determine their relative significance. To present the numerical results of our calculations we use Tab. 2. We present our results as matrix (9). All comparisons are done on the basis of subjective assessments.

Below is one comparison provided as an example: the optimal use of equipment has a much higher significance than the available communications and the utility lines. Therefore, we place digit 5 at the intersection of a line with optimal use of equipment and a column with communications/utility lines and 1/5 at the intersection of the line of communications/utility lines and the column with the use of equipment.

As a result, we have a necessary matrix of paired comparisons (Tab. 4). When the matrix has been built, we can calculate the priority values for each criterion using formula (11). The calculation results are shown in Tab. 4.

After the calculations have been done, we have to make sure that the condition of the consistency in the paired comparison matrix is met, for which purpose we use formulas (12) and (13). The calculation results demonstrate that the matrix is consistent. This means that the obtained values of criteria priorities can be used for further computations.

Now let us define local priorities of investment projects in relation to the criteria. For this purpose, we build matrixes of criteria value comparisons for investment projects 1 and 2. This means that we have to determine the degree of compliance of each criterion with the other ones. This assessment has a subjective nature, as in the case of criteria comparison. For instance, ten houses are being built on site 1, and they are

Table 4

The matrix of paired comparisons of priority criteria and the column with values and priorities of the criteria

	Optimal use of construction equipment	Communications	Availability of electric power substation	Infrastructure	Environmental situation	Priority of the investments object related to criterion w_{ij}
Optimal use of construction equipment	1	5	5	7	9	0.549
Communications	1/5	1	3	5	7	0.231
Availability of electric power substation	1/5	1/3	1	3	5	0.126
Infrastructure	1/7	1/5	1/3	1	3	0.062
nvironmental situation	1/9	1/7	1/5	1/3	1	0.032

located close to each other. This allows us to use only one pillar crane mounted on rails for the construction of all the houses. Communications and utility lines on the site are inexistent and, therefore, we have to lay and construct them. Thus we write digit 9 at the intersection of the line with optimal use of equipment and the column with communications/utility lines, and 1/9 at the intersection of the string of communications/utility lines and the column with the use of equipment.

Using formula (11), we calculate local investment project 1 priorities for each criterion w_{ij}. Here *i* refers to the serial number of an object while *j* is the serial number of the criterion. The calculation results are shown in Tab. 5.

After the calculations have been done, we have to make sure that the condition of consistency in the matrix of local priorities of the investment project is met, for which purpose we use formulas (12) and (13). The calculation results demonstrate that the matrix is consistent.

Having obtained the criteria priorities (Tab. 5) and the local priorities of the investment project according to these criteria, we can carry out a synthesis of the criteria using formula (14) and define the integral priority w* of object 1. As a result, we get the value w1* = 0.380.

Now we can do the similar calculations for investment project 2 and present them in Tab. 6.

Table 5

The matrix of local priorities of investment project 1 in relation to the criteria, and local priorities of object 1 according the criteria

	Optimal use of construction equipment	Communications	Availability of electric power substation	Infrastructure	Environmental situation	Priority of the investments object related to criterion w_{ij}
Optimal use of construction equipment	1	9	5	7	7	0.604
Communications	1/9	1	1	1	1	0.083
Availability of electric power substation	1/5	1	1	5	5	0.179
Infrastructure	1/7	1	1/5	1	5	0.088
Environmental situation	1/7	1	1/5	1/5	1	0.046

Table 6

The matrix of local priorities of investment project in the town of Lomonosov, and local priorities of object 2 according the criteria

	Optimal use of construction equipment	Communications	Availability of electric power substation	Infrastructure	Environmental situation	Priority of the investments object related to criterion w_{ij}
Optimal use of construction equipment	1	2	4	4	4	0.366
Communications	1/2	1	7	7	7	0.388
Availability of electric power substation	1/4	1/7	1	5	6	0.141
Infrastructure	1/4	1/7	1/5	1	3	0.064
Environmental situation	1/4	1/7	1/6	1/3	1	0.040

We see that the matrix is consistent, and we do calculations of the integral priority of object w^* using formula (14). The resulting value is the integral priority for object 2: $w2^* = 0.314$.

Having defined the integral priorities of investment objects 1 and 2, we can determine a higher priority object by applying a simple comparison: $w1^* > w2^*$ because $0.380 > 0.314$.

The comparison demonstrates a considerably higher priority of investment project 1 over investment project 2w. Therefore, *project 1 is chosen for further implementation*.

Conclusion. The current context of the regional investment market must facilitate the determination of investments' effectiveness according to their priority level, which requires the calculation of the value of a priority criterion. This is particularly important in a situation of financing deficit. The economic mathematical model presented above helps choose a more advantageous project during investment-related decision making. The chief goal is the organization of the objects according to the selected priority criteria. The proposed approach, reflected in the economic mathematical model, helps built an optimal investment program of a region in the context of a financing deficit.

REFERENCES

1. **Rastrigin L.A.** Contemporary principles to control complex objects [Sovremennyye printsipy upravleniya slozhnymi obyektami]. Moscow, Sov. radio, 1980. 286 p. (rus)
2. **Durov Ju.A., Travkin S.I., Yakimets, V.N.** Mnogokriterialnyye modeli formirovaniya i vybora variantov sistem [Multicriteria models of formation and choice of variants of systems]. Moscow, Science, 1986. 275 c. (rus)
3. **Chernoruzki I.G.** Metody optimizatsii i prinyatiya resheniy [Optimization and decision making methods]. St. Petersburg, Lan, 2001. 384 p. (rus)
4. **Saati T.** Mathematical models of conflict: Translation of Engl. Moscow, Sov. radio, 1977. 304 p.
5. **Hamdy A. Taha.** Operations research: an introduction, 7nd ed.: D from English. Moscow, Williams, 2005. 912 p.

СПИСОК ЛИТЕРАТУРЫ

1. **Растригин, Л.А.** Современные принципы управления сложными объектами [Текст] / Л.А. Растригин. — М.: Сов. радио, 1980. — 286 с.
2. **Дуров, Ю.А.** Многокритериальные модели формирования и выбора вариантов систем [Текст] / Ю.А. Дуров, С.И. Травкин, В.Н. Якимец. — М.: Наука, 1986. — 275 с.
3. **Черноруцкий, И.Г.** Методы оптимизации и принятия решений [Текст] / И.Г. Черноруцкий. — СПб.: Лань, 2001. — 384 с.
4. **Саати, Т.** Математические модели конфликтных ситуаций [Текст] : пер. с англ. / Т. Саати. — М.: Сов. радио, 1977. — 304 с.
5. **Хемди, А.Т.** Введение в исследование операций [Текст] : пер. с англ. / А.Т. Хемди. — 7-е изд. — М.: Изд. дом «Вильямс», 2005. — 912 с.

NIKOLOVA, Liudmila V. — *Saint-Petersburg State Polytechnical University.*
195251, Politekhnicheskaya str. 29. St. Petersburg. Russia. E-mail: nikalvsk@yandex.ru

НИКОЛОВА Людмила Васильевна- *профессор кафедры финансов и денежного обращения Инженерно-экономического института Санкт-Петербургского государственного политехнического университета, доктор экономических наук, профессор.*
195251, Россия, Санкт-Петербург, ул. Политехническая, д. 29. E-mail: nikalvsk@yandex.ru

RADIONOVA, Julia V. — *Saint-Petersburg State Polytechnical University.*

РАДИОНОВА Юлия Владимировна — *аспирант кафедры финансов и денежного обращения Инженерно-экономического института Санкт-Петербургского государственного политехнического университета.*
195251, Россия, Санкт-Петербург, ул. Политехническая, д. 29. E-mail: conly@mail.ru

© St. Petersburg State Polytechnical University, 2013